Assignment 4
MATH414
Statistics

QUESTIONS

(1) (Hogg, McKean, Craig, pp360) Let $X_1, X_2, X_3, \ldots, X_n$ represent a random sample from each of the distributions having the following pdfs:

   (a) $f(x, \theta) = \theta x^{\theta-1}, 0 < x < 1, \theta \in (0, \infty)$, zero elsewhere.

   (b) $f(x, \theta) = e^{-(x-\theta)}, \theta \leq x < \infty, \theta \in \mathbb{R}$, zero everywhere.

In each case, find the mle $\hat{\theta}$ of θ.

(2) (Hogg, McKean, Craig, pp361) Suppose $X_1, X_2, X_3, \ldots, X_n$ are iid with pdf $f(x, \theta) = 2x/\theta^2, 0 < x < \infty$, zero elsewhere. Find:

   (a) the mle $\hat{\theta}$ of θ.

   (b) The constant c so that $E(c\hat{\theta}) = \theta$.

(3) (Hogg, McKean, Craig, pp373) Prove that $\bar{X}$, the mean of a random sample of size n from a distribution that is $N(\theta, \sigma^2)$, $-\infty < \theta < \infty$, is, for every known $\sigma^2 > -$, an efficient estimator of θ.

(4) Let $Y_1, Y_2, \ldots, Y_n$ be independent and identically distributed rvs distributed as $\Gamma(1, 1/\theta)$. Let $W = \sum_{i=1}^{n} Y_i$. Compute the expectation of $1/W$ and $1/W^2$.

(5) (Hogg, McKean, Craig, pp376) Let S^2 be the sample variance of a random sample of size $n > 1$ from $N(\mu, \theta)$, $0 < \theta < \infty$, where μ is known. We know $E(S^2) = \theta$.

    (a) What is the efficiency of S^2?

    (b) What is the mle $\hat{\theta}$ of θ?

    (c) What is the asymptotic distribution of $\sqrt{n}(\hat{\theta} - \theta)$?

(6) (Hogg, McKean, Craig, pp374-5) Suppose $X_1, X_2, X_3, \ldots, X_n$ are iid with $\Gamma(4, \theta)$ distribution, $0 < \theta < \infty$.

    (a) What is the Fisher information $I(\theta)$?

    (b) What is the mle $\hat{\theta}$ of θ?

    (c) What is the asymptotic distribution of $\sqrt{n}(\hat{\theta} - \theta)$?