

Assignment 4

MATH414

Statistics

INSTRUCTIONS

- Write your assignments with a pen on paper and submit it in the physical form.
 - The use of ChatGPT or any other generative AI or, in fact, any “calculator” such as “WolframAlpha” is strictly forbidden. If you use any of these, you will get zero in the entire Assignment. The exception to this rule is when you want to check a calculation once you have performed it.
 - You are welcome to look up any theory or definitions from any source you like but you must cite it.
 - Write your answers cleanly.
 - You need to fully write the R commands that you use.
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QUESTIONS

- (1) (Hogg, McKean, Craig, pp360) Let $X_1, X_2, X_3, \dots, X_n$ represent a random sample from each of the distributions having the following pdfs:
- (a) $f(x, \theta) = \theta x^{\theta-1}$, $0 < x < 1$, $\theta \in (0, \infty)$, zero elsewhere.
 - (b) $f(x, \theta) = e^{-(x-\theta)}$, $\theta \leq x < \infty$, $\theta \in \mathbb{R}$, zero everywhere.
- In each case, find the mle $\hat{\theta}$ of θ .
- (2) (Hogg, McKean, Craig, pp361) Suppose $X_1, X_2, X_3, \dots, X_n$ are iid with pdf $f(x, \theta) = 2x/\theta^2$, $0 < x < \infty$, zero elsewhere. Find:
- (a) the mle $\hat{\theta}$ of θ .
 - (b) The constant c so that $E(c\hat{\theta}) = \theta$.
- (3) (Hogg, McKean, Craig, pp373) Prove that \bar{X} , the mean of a random sample of size n from a distribution that is $N(\theta, \sigma^2)$, $-\infty < \theta < \infty$, is, for every known $\sigma^2 > 0$, an efficient estimator of θ .
- (4) Let Y_1, Y_2, \dots, Y_n be independent and identically distributed rvs distributed as $\Gamma(1, 1/\theta)$. Let $W = \sum_{i=1}^n Y_i$. Compute the expectation of $1/W$ and $1/W^2$.

- (5) (Hogg, McKean, Craig, pp376) Let S^2 be the sample variance of a random sample of size $n > 1$ from $N(\mu, \theta)$, $0 < \theta < \infty$, where μ is known. We know $E(S^2) = \theta$.
- (a) What is the efficiency of S^2 ?
 - (b) What is the mle $\hat{\theta}$ of θ ?
 - (c) What is the asymptotic distribution of $\sqrt{n}(\hat{\theta} - \theta)$?
- (6) (Hogg, McKean, Craig, pp374-5) Suppose $X_1, X_2, X_3, \dots, X_n$ are iid with $\Gamma(4, \theta)$ distribution, $0 < \theta < \infty$.
- (a) What is the Fisher information $I(\theta)$?
 - (b) What is the mle $\hat{\theta}$ of θ ?
 - (c) What is the asymptotic distribution of $\sqrt{n}(\hat{\theta} - \theta)$?