Assignment 2
MATH414
Statistics

INSTRUCTIONS

- Write your assignments with a pen on paper and submit it in the physical form.
- The use of ChatGPT or any other generative AI or, in fact, any "calculator" such as "WolframAlpha" is strictly forbidden. If you use any of these, you will get zero in the entire Assignment. The exception to this rule is when you want to check a calculation once you have performed it.
- You are welcome to look up any theory or definitions from any source you like but you must cite it.
- Write your answers cleanly.
- You need to fully write the R commands that you use.

QUESTIONS

(1) (Hogg, McKean & Craig, pp171) The mgf of a random variable X is $e^{4(e^t-1)}$. Show that $P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.931$.

(2) (Hogg, McKean & Craig, pp182) Let X and Y have the joint pmf $p(x,y) = e^{-2}/[x!(y-x)!]$, $y = 0,1,2,\ldots$, $x = 0,1,2,\ldots,y$, zero elsewhere.
  (a) Find the mgf $M(t_1,t_2)$ of the joint distribution.
  (b) Compute the means, the variances, and the correlation coefficient of X and Y.
  (c) Determine the conditional mean $E[X|y]$. (*Hint*: Show that $\sum_{x=0}^{y}[\exp(t_1 x)]y!/[x!(y-x)!] = [1+\exp(t_1)]^y$).

(3) (Hogg, McKean & Craig, pp182) Suppose $(1-2t)^{-6}$, $t < \frac{1}{2}$ is the mgf of the random variable X.
  (a) Identify the distribution of X.
  (b) Find out the R command for cumulative distribution function for the distribution in part a, and compute $P(X < 5.23)$.
  (c) Find the mean μ and variance σ^2 of X. Use R to compute $P(|X-\mu| < 2\sigma)$.

(4) (Hogg, McKean & Craig, pp183) Show that

$$\int_{\mu}^{\infty} \frac{1}{\Gamma(k)} z^{k-1} e^{-z}\, dz = \sum_{x=0}^{k-1} \frac{\mu^x e^{-\mu}}{x!},\ k = 1,2,3,\ldots.$$

Look at the hint given in the text. Also, mention if there are mathematical results that you would require for this computation to be correct such as interchange of limits, integrals, sums etc.

(5) (Hogg, McKean & Craig, pp194) If X is $N(75, 100)$, find $P(X < 60)$ and $P(70 < X < 100)$ by using the R command `pnorm`.

(6) (Hogg, McKean & Craig, pp194) If X is $N(\mu, \sigma^2)$, find b so that $P[-b < (X - \mu)/\sigma < b] = 0.90$, by using the R command `qnorm`.

(7) (Hogg, McKean & Craig, pp197) Let X_1 and X_2 be independent with normal distributions $N(6, 1)$ and $N(7, 1)$, respectively. Find $P(X_1 > X_2)$. Hint: Write $P(X_1 > X_2) = P(X_1 - X_2 > 0)$ and determine the distribution of $X_1 - X_2$.

(8) (Hogg, McKean & Craig, pp216) Let T have a t-distribution with 10 degrees of freedom. Find $P(|T| > 2.228)$ by using R.

(9) (Hogg, McKean & Craig, pp218) Let the random variable W have a F-distribution with parameters r_1 and r_2. Show that $Y = \frac{1}{1+(r_1/r_2)W}$ has a beta distribution.

(10) (Hogg, McKean & Craig, pp218) Let X_1, X_2 and X_3 be three independent chi-square variables with r_1, r_2 and r_3 degrees of freedom, respectively.

(a) Show that $Y_1 = X_1/X_2$ and $Y_2 = X_1 + X_2$ are independent and that Y_2 is $\chi^2(r_1 + r_2)$.

(b) Deduce that

$$\frac{X_1/r_1}{X_2/r_2} \text{ and } \frac{X_3/r_3}{(X_1 + X_2)/(r_1 + r_2)}$$

are independent F-variables.

(11) (Hogg, McKean & Craig, pp217) Let F have an F-distribution with parameters r_1 and r_2 . Argue that $1/F$ has an F-distribution with parameters r_2 and r_1 .